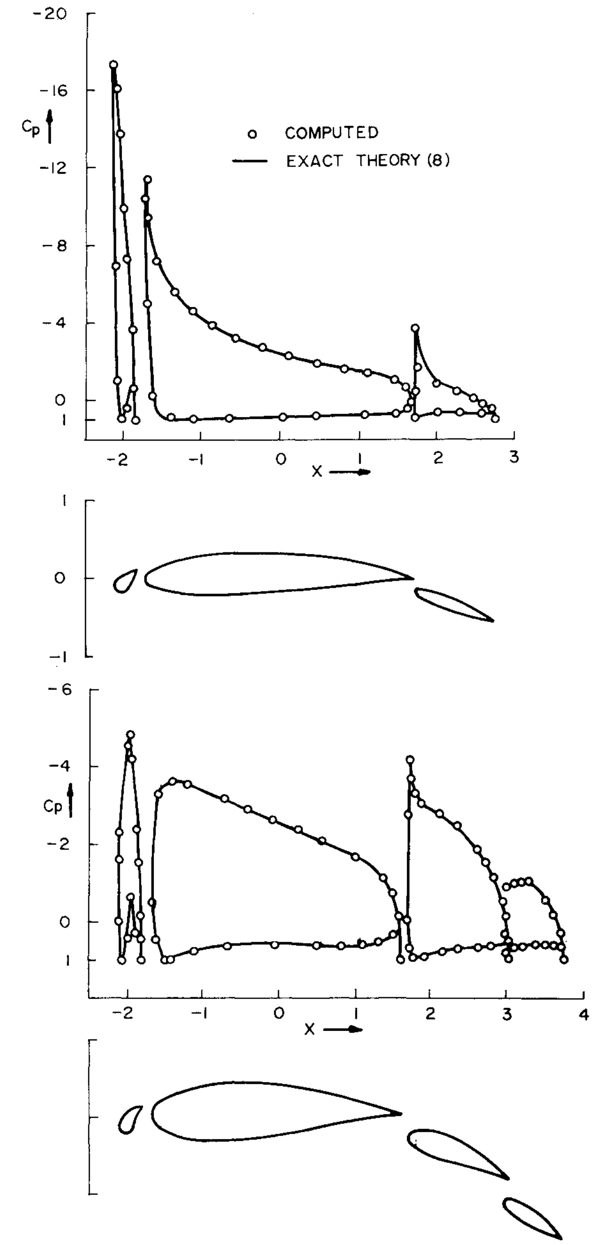

Fig. 4 Comparison of C_p distributions for three- and four-element airfoils.

Conclusions

The distributed vortex singularity method using flat panels with linearly varying vortex strength for obtaining inviscid solution on single or multi-element airfoils is an accurate and stable method. Once the limiting behavior of trailing-edge panels is correctly accounted for, the method can analyze airfoils of arbitrary thickness and camber, airfoils with finite trailing-edge angle, and airfoils with cusped trailing edges. Also, there are no problems faced in analyzing extremely thin airfoils. Another important advantage of the current formulation is that the pressure coefficients are computed at the contour input points, whereas most methods output the C_P at panel midpoints, which in fact are not points on the contour. The method has been checked for a very large number of cases, and the results show that as few as 20 panels are enough to compute pressure coefficients over a single-component airfoil. A viscous version of the method is available in Ref. 9.

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Unsteady Pressure Distribution Over a Pitching Airfoil

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Introduction

THE problem of unsteady aerodynamic loads over airfoils has been intensively studied, but some published contradictory results still raise the question of whether a potential model is valid or not. The general frame of the present investigation is the unsteady loads over airplane wings during flight in turbulence. The main unsteady parameter is the reduced frequency, $k = 2\pi fc/U_\infty$, and in the present case the range of interest is $0<k<1.2$. The amplitude of the angle-of-attack fluctuations is small enough to ensure that the flow remains attached, at least over the main part of the airfoil. However, the unsteady mechanism of the separation of the boundary layers at the trailing edge, and consequently the modeling of the Kutta condition in inviscid numerical codes, remains an open subject.

A lot of experimental work has been done concerning a possible violation of the Kutta condition.[1-4] As pointed out by Telionis and Poling,[4] most of these investigations examine pressure distributions, especially the differential pressure loading at the trailing edge. Satyanarayana and Davis[1] observe a nonzero loading (first harmonic of the unsteady pressure) at $k = 1.23$ on a pitching airfoil. They conclude that the failure of the Kutta condition is obvious.

This result seems questionable. First, the differential pressure loading must vanish physically at the trailing edge, at least in a real viscous fluid. The question is rather the rate at

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which $\Delta p(x/c)$ decreases as x/c tends to 1. Second, it must be remembered that the Kutta condition, either steady or unsteady, is nothing other than a way to render the inviscid solution unique, assuming that separation of the boundary layer takes place at the trailing edge.[5] In a viscous-inviscid matching process,[6,7] the Kutta condition introduces viscous effects early in the 0(1) "outer" solution.[8] Hence, it is more convenient to speak of a failure of the inviscid solution than of a failure of the Kutta condition.

When dealing with the unsteady phenomena, one may question either the mathematical aspects of the Kutta condition (see Basu and Hancock for a clean presentation[9]) or the practical limitations of an inviscid modeling of the flow. In the present work, a two-dimensional nonlinear potential computation has been compared to experimental data obtained over a symmetrical pitching airfoil in a subsonic wind tunnel. The range of applicability of a potential computation is discussed.

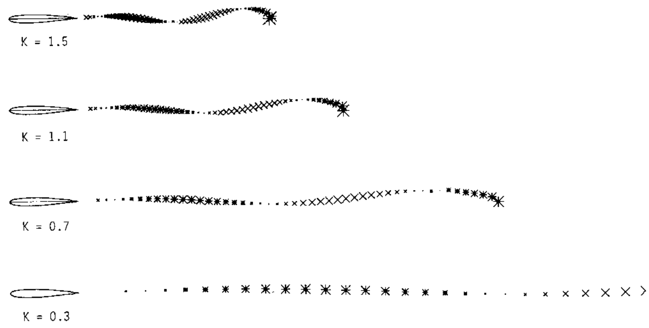

Fig. 1 Computed vortex patterns as a function of the reduced frequency k; the size of the symbols is proportional to the vortex intensity, * = clockwise rotating vortex, + = anticlockwise rotating vortex.

Computational Method

The method of computation follows the two-dimensional procedure described by Basu and Hancock.[9] The airfoil, a NACA 0012, is divided into N segments ($N = 40$) supporting sources or sinks of constant value $\sigma_I(t)$ on each segment and a constant vortex density over the whole surface:

$$\Gamma(t) = \int_A \gamma(t)\, \mathrm{d}s = \gamma(t) \tag{1}$$

where Γ is the instantaneous circulation, γ the vortex density, and P the perimeter of the airfoil (≥ 2 chords).

To compute the values of the $N + 1$ unknowns [$\sigma_I(t)$, $I = 1$ to N, and $\gamma(t)$], N linear relations arise from the zero normal relative velocity at the center of each segment:

$$[V_F(t) - V_s(t)] \cdot n(t) = 0 \tag{2}$$

where V_F is the fluid velocity, V_s the surface velocity, and n the normal at the surface.

The $(N + 1)$th relation is the Kutta condition. Following Sears[10] or Basu and Hancock,[9] the zero differential loading at the trailing edge is

$$\Delta p(t)|_{TE} = P_u - P_L \tag{3}$$

where u and L designate the upper and lower limit of the airfoil ($x/c \to 1$) and may be transformed by means of the unsteady Bernoulli equation into

$$2 \cdot \frac{\partial}{\partial t}(\Phi_u - \Phi_L) - (V_u^2 - V_L^2) = 0 \tag{4}$$

or

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = -\tfrac{1}{2}(V_u^2 - V_L^2) \tag{5}$$

Equation (5) is consistent with the model of vortex shedding. The variations of Γ are balanced by the emission of a vortex of intensity $(V_u - V_L)$ at a mean velocity $(V_u + V_L)/2$. Variables V_L and V_u are the limiting values of the velocity at the surface as $x/c \to 1$. They are numerically unattainable since the derivative $\partial V/\partial s$ is singular at the trailing edge, where a stagnation value should be found on one of the two faces.[9,11] In the present computation, V_L and V_u are estimated from an extrapolation of the velocities computed at the center of the last segments.

The wake is represented by a straight segment attached to the trailing edge for the current time step. The length and orientation is given by

$$\Delta l = \tfrac{1}{2}(V_u + V_L)\, \mathrm{d}t \tag{6}$$

and the vorticity carried by the segment is simply

$$\Delta\Gamma = \gamma_{TE} \cdot \mathrm{d}l = -\frac{\mathrm{d}\Gamma}{\mathrm{d}t}\Delta t \tag{7}$$

The problem is nonlinear, and the solution has to be found iteratively at each time step until Eq. (5) is satisfied. A partial linearization is made, however. The wake is regarded as known (except the *TE* element) at the current time step. When the solution is found, the *TE* element is transformed into a discrete vortex, and every vortex is convected according to the velocity induced at its position by every other singularity. The typical vortex pattern is presented in Fig. 1 for a $\pm$1-deg-amplitude sinusoidal pitching notion.

Experimental Apparatus

The experiments were conducted in a 1.00×1.20-m^2 subsonic wind tunnel ($0 < V < 60$ m/s). The rectangular wing (chord = 0.40 m, span = 1.00 m) is made of composite materials and a steel axle mounted on ball bearings outside of the tunnel. The axis of rotation is located at 25% of the chord. The pitching motion is generated by an electrical motor actuating a connecting rod and crank setting.

The maximum frequency is restricted to 30 Hz due to a mechanical resonance occurring at 40 Hz. The amplitude was fixed at 2 deg *p-p* for the results presented in this paper.

The center section of the wing is equipped with 30 pressure ports connected to a multichannel piezoresistive pressure transducer (P.S.I. model ESP 32). The last pressure ports are located at 96.5 and 98.5% of the chord to obtain data very close to the trailing edge. To minimize the tubing length (0.35 m, $\phi = 1.5$ mm), the pressure transducer is located inside the wing, close to the axis of rotation, in order to eliminate a parasitic response of the transducer to the acceleration. The time response of the setup (port + tubing + transducer) has been estimated, given the hypothesis of a viscous capillary-type flow inside the tube.[12] The computed time to reach 99% of the final value after a step is 0.6 ms. In fact, the tube behaves rather than a second-order system, and an experimental checking has shown the actual time lag to be 0.5 ms and the amplitude ratio (measured pressure/input pressure) 1.06 at 30 Hz ($k \simeq 1.5$). Data were sampled and processed by a DEC PDP 11/23 computer.

Results

Careful attention was paid to several parameters (Reynolds number, amplitude effects) in order to ensure that none of these parameters had a determinant influence. The boundary layers are tripped artificially to discard some transitional instabilities from the unsteady data. The experiments were performed at a Reynolds number of 0.6×10^6 (22.5 m/s), and the amplitude was set to 2 deg *p-p* after checking that the amplitude-pressure relation was linear in that range of incidence fluctuations. The comparison between the computed and mea-

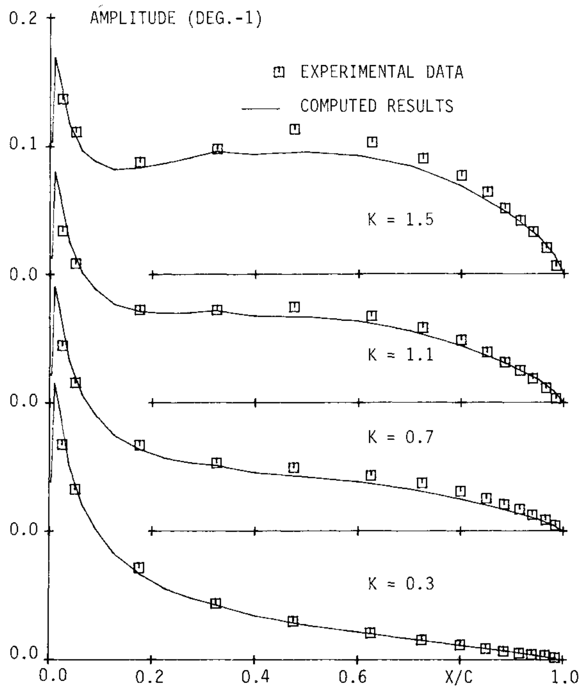

Fig. 2 Comparison between the measured and computed amplitude of the surface-pressure fluctuations. The unsteady pressure coefficient K_p is divided by the amplitude of the angular fluctuation (1 deg).

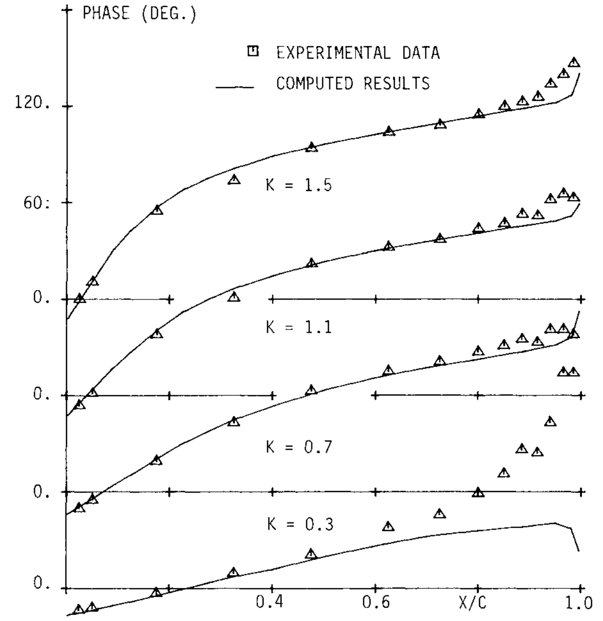

Fig. 3 Comparison between the measured and computed phases of the surface pressure fluctuations.

sured amplitude and phase of the first harmonic of the unsteady pressure are presented in Figs. 2 and 3 for several values of the reduced frequency k.

The general agreement is good, except for the phase in the trailing-edge region at the lower reduced frequency. The experimental determination of the phase seems questionable, since the amplitude is very small and the signal becomes "noisy" due to turbulent fluctuations. Another discrepancy appears at $k = 1.5$ in the amplitude comparison. Here, a bending of the wing occurs due to the high frequency, and a plunging motion is superposed to the pitching motion.

Discussion and Conclusion

The overall agreement between the measured and computed pressure fluctuations over a pitching airfoil seems to validate the inviscid model in a range of reduced frequencies extending up to $k = 1.5$. The predicted loading of the trailing-edge region agrees well with the measured values, and no spectacular deviation appears. The deviation between the theoretical and experimental data mentioned by Ref. 1 has not been verified, either for the amplitude or the phase comparisons. Although the wing section is different (NACA 64A010 in Ref. 1), the others parameters (amplitude, Reynolds number, reduced frequency) are identical.

The question of whether the Kutta condition is verified or not depends on the exact meaning given to this condition. As far as the differential pressure loading in the trailing edge region is concerned, the condition is verified, at least in the range of amplitude and reduced frequencies that were investigated. However, the phenomenological behavior of the velocity as described by Maskell[9] cannot be proven, either experimentally, since the flow is viscous, or numerically, due to the singularity of the solution at the trailing edge in the physical plane.

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Coupled Thermoelasticity Beam Problems

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Introduction

RAPID changes in thermal gradient with respect to time of an elastic continuum requires that the first law of thermodynamics include rate effects; therefore, the solution for tem-

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